Financial "Anti-Bubbles": Log-Periodicity in Gold and Nikkei collapses

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February 1, 2008

Abstract

We propose that imitation between traders and their herding behaviour not only lead to speculative bubbles with accelerating over-valuations of financial markets possibly followed by crashes, but also to "anti-bubbles" with decelerating market devaluations following all-time highs. For this, we propose a simple market dynamics model in which the demand decreases slowly with barriers that progressively quench in, leading to a power law decay of the market price decorated by decelerating log-periodic oscillations. We document this behaviour on the Japanese Nikkei stock index from 1990 to present and on the Gold future prices after 1980, both after their all-time highs. We perform simultaneously a parametric and non-parametric analysis that are fully consistent with each other. We extend the parametric approach to the next order of perturbation, comparing the log-periodic fits with one, two and three log-frequencies, the latter one providing a prediction for the general trend in the coming years. The non-parametric power spectrum analysis shows the existence of log-periodicity with high statistical significance, with a prefered scale ratio of $\lambda \approx 3.5$ for the Nikkei index $\lambda \approx 1.9$ for the Gold future prices, comparable to the values obtained for speculative bubbles leading to crashes.

Pacs numbers: 01.75+m ; 02.50-r ; 89.90+n

1 Introduction

Since our first suggestion of the existence of specific log-periodic price patterns associated to the speculative accelerating power law bubble leading to the worldwide Oct. 1987 stock market crash [1], more evidences have accumulated [1-11]. They include the Oct. 1929 US crash [2, 3], the May 1962 US market correction of 15% discovered in a systematic blind-testing procedure [4, 5], the Hong-Kong Oct. 1997 crash [4, 5], the Oct. 1997 US market correction [6-9] (see also the prediction communicated on sept. 17, 1997 to the French office for the protection of proprietary softwares and inventions under number registration 94781 and footnote 13 of Ref.[10]) and the Aug. 1998 US market correction of 19% [11].

Recently, the same log-periodic signatures with similar prefered scaling ratios and power law exponents have been found to describe the speculative behaviour in the Reagan's years of the US dollar against other major currencies, such as the German Mark and the Swiss Franc up to the maximum of the bubble in March 1985 [11]. It is remarkable to find that very similar structures describe both the behaviour of entire markets quantified by their indices and the behaviour of the Foreign exchange (Forex). The two are generally believed to obey different dynamics [12], since on the Forex only a few major banks perform the trading and only a few currencies (the US$ against the DEM now replaced by the Euro and the YEN) account for most (more than 80%) of the trading.

This strengthens the concept that these structure are characteristic of very fundamental and robust properties of financial markets, which we have proposed to be the herding behaviour of traders [4, 11, 13]. Furthermore, it supports the view according to which the stock markets present self-organising properties similar to those of microscopic models in statistical physics, with the potential for cooperative behaviour [14] leading to critical points [9, 15] (for criticism, see [16]).

In addition, these results suggest that the prefered scale ratio λ of the underlying discrete scale invariance [17] of the market dynamics is remarkably robust with respect to changes in *what* is traded and *when* it is traded. These results are at odds with the standard random walk null-hypothesis and the efficient market hypothesis. This hypothesis leads to the conclusion that stock and currency markets are unpredictable.

The question we address here is whether the cooperative herding behaviour of traders might also produce market evolutions that are symmetric to the accelerating speculative bubbles often ending in crashes. This symmetry is performed with respect to a time inversion around a critical time t_c such that $t_c - t$ for $t < t_c$ is changed into $t - t_c$ for $t > t_c$. This symmetry implies to look at *decelerating* devaluations instead of accelerating valuations. A related observation has been reported in our first paper [1] showing that the implied volatility of traded options, which is a measure of the perceived market risk, has relaxed *after* the Oct. 1987 crash to its long-term value, from a maximum at the time of the crash, over a time-scale of about a year, according to a decaying power law with decelerating log-periodic oscillations. It is this type of behaviour that we document now but for real prices.

We show that there seems to exist critical times t_c at which the market culminates, with either a power law increase with accelerating log-periodic oscillations or a power law decrease with decelerating log-periodic oscillations. We have not found a market for which both phenomena are simultaneously observed for the same t_c. The main reason is that accelerating markets with log-periodicity almost always end-up in a crash, a market rupture that thus breaks down the symmetry ($t_c - t$ for $t < t_c$ into $t - t_c$ for $t > t_c$). The main message

that we draw here is that herding behaviour can progressively occur and strengthen itself in "bearish" (decreasing) market phases, even if the preceding "bullish" phase ending at t_c was not characterised by a strengthening imitation. The symmetry is thus statistical or global and holds in the ensemble rather than for each single case individually. The breakdown of local symmetry around the critical point t_c is not unknown in thermodynamic phase transitions. Let us mention the $\lambda-$transition in ^4He, so named because of the asymmetric shape of the specific heat around T_λ with a more abrupt decay above T_λ than below [18] qualitatively similar to a market price time series around a crash.

The organisation of the paper is as follows. We first present the Landau expansion and its extension to third-order. The three derived log-periodic formulas are used to fit the Japanese Nikkei index drop since 1990. They agree very well in the domains where they overlap, not only in their shape but also in the coherence of their parameters. The new third-order formula gives an excellent fit up to the end of 1998 and provides a prediction for the general trend of 1999 and 2000. Further empirical evidence is presented on the Gold devaluation a few months after its peak in 1980 over a period of more than two years. From parametric fits with log-periodic formulas and a non-parametric log-periodic power spectrum analysis, we find strong evidence of structures very similar to those found previously prior to crashes. We then propose a simple dynamical model of imitative "bearish" market, which using the empirical tests, lead to the prediction that the relative strength of the optimistic phase in an otherwise pessimistic bearish market is stronger than the occasional pessimism in an otherwise speculative bullish bubble.

2 Data analysis: Nikkei and Gold

2.1 Landau expansion and log-periodic formulas

A very powerful and general tool used in the studies of critical phase transitions is that of Landau expansions. They amount to assuming some functional relationship $F(\tau)$ between the relevant observable F and the corresponding governing parameter τ. With respect to the financial markets, the observable can be the price (or some measure of it) and the governing parameter is time (or some measure of it). In Ref.[3], we have proposed a general form for an evolution equation for the complex variable F, obtained by expanding F around 0 as

$$\frac{d \log F(\tau)}{d \log \tau} = \alpha F(\tau) + \beta |F(\tau)|^2 F(\tau) \ldots , \tag{1}$$

where in general the coefficients may be complex. In this expansion, only terms that ensure that the equation remains invariant with respect to a change of phase $\phi \to \phi + C$ where

$$F \equiv B e^{i\phi} \tag{2}$$

are allowed since a phase translation corresponds to a change of time units [3].

A first (keeping only the $\alpha F(\tau)$ term in the r.h.s. of (1)) and second order solution (keeping the two first terms in the r.h.s. of (1)) gives the following two equations for the price (or some measure of it) evolution [3]

$$p(t) \quad \approx \quad A + B\tau^\alpha + C\tau^\alpha \cos\left[\omega \log(\tau) + \phi\right] \tag{3}$$

$$p(t) \approx A' + \frac{\tau^\alpha}{\sqrt{1+\left(\frac{\tau}{\Delta_t}\right)^{2\alpha}}} \left\{ B' + C' \cos\left[\omega \log\tau + \frac{\Delta_\omega}{2\alpha}\left(1+\left(\frac{\tau}{\Delta_t}\right)^{2\alpha}\right) + \phi'\right]\right\}, \quad (4)$$

where $\tau = t_c - t$. The log-periodic frequency ω is related to the prefered scaling ratio by

$$\ln\lambda = \frac{2\pi}{\omega} \ . \quad (5)$$

These two equations (3,4) describe the evolution of the price *prior* to a time t_c, where a large crash may occur, i.e., $t < t_c$. Equation 3 have been found to describe the price evolution up to 3 years prior to large crashes [11] and eq. 4 up to 8 years [3]. Observe that eq.(4) predicts the transition from the log-frequency ω close to t_c to $\omega + \Delta_\omega$ far from t_c (i.e. for $\tau > \Delta_t$).

From the point of view of critical points and within Landau expansions, there is nothing special about the direction of the "flow" or equivalently the sign of τ in eq. 1. We thus propose to use equations (3,4) replacing τ with $-\tau$ (or $t_c - t$ with $t - t_c$), where now $t > t_c$. This novel situation corresponds to the case of a (power law) decaying price, i.e., an "anti-'bubble" or, in financial colloquial terms, a "bearish" phase.

We will use below an extension of (3,4) obtained from the Landau equation (1) expanded up to the next order

$$\frac{d\log F(\tau)}{d\log\tau} = \alpha F(\tau) + \beta |F(\tau)|^2 F(\tau) + \gamma |F(\tau)|^4 F(\tau) \dots \ . \quad (6)$$

Inserting (2) into (6) gives

$$\frac{dB}{d\log\tau} = a_1 B + b_1 |B|^2 B + c_1 |B|^4 B \ , \quad (7)$$

$$\frac{d\phi}{d\log\tau} = a_2 + b_2 |B|^2 + c_2 |B|^4 \ , \quad (8)$$

where

$$\alpha = a_1 + i a_2 \ , \quad (9)$$

$$\beta = b_1 + i b_2 \ , \quad (10)$$

$$\alpha = c_1 + i c_2 \ . \quad (11)$$

The solution of (7) is given by the implicit equation

$$\frac{\left[\frac{B^2 - B_+^2}{B^2}\right]^{B_-^2}}{\left[\frac{B^2 - B_-^2}{B^2}\right]^{B_+^2}} = \left(\frac{\tau}{\tau_0}\right)^{2c_1 B_-^2 B_+^2 (B_+^2 - B_-^2)} \ , \quad (12)$$

where

$$B_\pm^2 = \frac{1}{2c_1}\left(-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}\right) \ . \quad (13)$$

Certain conditions must hold for a solution to exist, in particular $b_1/c_1 < 0$ to ensure that the first non-linear correction leads to another stable fixed point and $b_1^2 \geq 4a_1c_1$ for B to be real.

Depending upon the values of the parameters, the solution $B(\tau)$ of (12) can take many different functional shapes. This leads in general to the problem of model misspecification. Here, for the purpose of simplicity, we choose the set of parameters such that the solution can be approximated by

$$p(t) \approx A' + \frac{\tau^\alpha}{\sqrt{1+\left(\frac{\tau}{\Delta_t}\right)^{2\alpha}+\left(\frac{\tau}{\Delta_t'}\right)^{4\alpha}}}$$

$$\left\{B' + C' \cos\left[\omega \log \tau + \frac{\Delta_\omega}{2\alpha} \ln\left(1+\left(\frac{\tau}{\Delta_t}\right)^{2\alpha}\right) + \frac{\Delta_\omega'}{4\alpha} \ln\left(1+\left(\frac{\tau}{\Delta_t'}\right)^{4\alpha}\right) + \phi\right]\right\} . \qquad (14)$$

Eq.(14) predicts the transition from the log-frequency ω close to t_c to $\omega+\Delta_\omega$ for $\Delta_t < \tau < \Delta_t'$ and to the log-frequency $\omega + \Delta_\omega + \Delta_\omega'$ for $\Delta_t' < \tau$. We stress that this corresponds to an *approximate* description of a log-frequency modulation and urge the reader to take the specific functional form of eq. (14) with some caution. The purpose here is simply to parameterise the data in order to acquire a prediction potential for the Nikkei.

2.2 Data analysis

2.2.1 The Nikkei "bearish" behaviour starting from 1st Jan. 1990

The most recent example of a genuine long-term depression comes from Japan, where the Nikkei has lost close to 60 % of its all-time high achieved on 31 Dec. 1989. In figure 1, we see the logarithm of the Nikkei from 31 Dec. 1989 until 31 dec. 1998. The fits are equations (3), (4) and (14) respectively with all nonlinear variables free for the two first equations and where the interval used for the first equation is until mid-1992 and for the second equation until mid-1995. Not only do the equations (3) and (4) agree remarkably well with respect to the parameter values produced by the fits, but they are also in good agreement with previous results obtained from stock market and Forex bubbles with respect to the values of exponent α [3, 11]. For the fit with (14), due to the large number of free variables, we performed differently. Of the 6 parameters t_c, α, Δ_t, ω, Δ_ω and ϕ' determined from the fit with (4) we kept the first 3 fixed and only Δ_t, Δ_t', Δ_ω, Δ_ω' and ϕ' where allowed to adjust freely. The results are given in the caption of figure 1. What lends credibility to the fit with eq. (14) is that despite it complex form, we get values for the two cross-over time scales Δ_t, Δ_t' which correspond very nicely to what is expected from the Landau expansion: Δ_t has moved down to 4.4 years, which is perfect with respect to the interval used for the two frequency formula (4) and Δ_t' is approximately 7.8 years, which is also fully compatible with the nine year interval of the fit. This does not mean that the cost-function space is not very degenerate, it is, but the ranking of Δ_t and Δ_t' is always the same and the values given does not deviate much from the ones in the caption of figure 3, i.e., by $\pm$ one unit.

The value obtained for $\omega \approx 4.9$ correspond to a scaling ratio $\lambda \approx 3.6$, which is significantly larger than the $2.2 < \lambda < 2.7$ previously obtained for the stock market [11]. An additional difference between the Nikkei and previous results is the strength of the oscillations compared to the leading behaviour. For the Nikkei, it is $\approx 20\%$, i.e., 2 to 3 times as large as the amplitude $\approx 5 - 10\%$ previously obtained for the stock market and the Forex [11]. We note that the fit with eq. (3) only produced the solution shown, whereas equations (4) and (14) produce multiple solutions. The solutions of equations (4) and (14) shown in figure 1 are the best solutions found which satisfies the criteria discussed in [4, 5].

2.2.2 The gold deflation price starting mid-1980

Another example of log-periodic decay is that of the Gold price after the burst of the bubble in 1980 as shown in figure 2. The bubble has an *average* power law acceleration as shown in the figure but *without* any visible log-periodic structure[1]. Again, we obtain a reasonable agreement with previous results for the exponent α but the value obtained for $\lambda \approx 1.9$ is now slightly lower than previously obtained. The strength of the oscillations compared to the leading behaviour is again $\approx 10\%$ as previously found. The fit with eq. (3) only produced the solution shown as for the Nikkei.

2.2.3 Non-parametric power spectrum analysis

In order to qualify further the significance of the log-periodic oscillations in a non-parametric way, we have eliminated the leading trend from the price data by the following transformation

$$p\left(t\right) \rightarrow \frac{p\left(t\right) - \left(A + B\tau^{\alpha}\right)}{C\tau^{\alpha}}. \tag{15}$$

In figure 3, we plot the remaining residue for the case of gold. We then analyse this residual structure by using a so-called Lomb periodogram, which is nothing but a power spectrum analysis using a series of local fits of a cosine (with a phase) with some user chosen range of frequencies. The advantage of the Lomb periodogram over a Fourier transform is that the points does not have to be equidistantly sampled. Applying this standard technique [19] to the two data sets shown in figures 1-3, we construct in figure 4 the power spectrum as a function of the log-frequency $f \equiv \omega/2\pi$ and find a peak at $f \approx 0.82$ for the Nikkei and $f \approx 1.59$ for the gold in good agreement with the fits of eq.s (3) and (4). The value $f = 0.82$ obtained from the periodogram of the Nikkei corresponds to $\lambda = 3.4$, which is significantly higher than previously obtained for crashes.

Since the nature of the "noise" is unknown, we cannot estimate the confidence interval of the peak in the standard way [19], but the relative level of the peak for each periodogram should be regarded a measure of the significance of the oscillations.

3 A simple dynamical model of "bearish" herding behaviour

We start from the formulation introduced in Ref.[20] relating the price variation to the net order size Ω over all traders, through a market impact function. Assuming that the ratio $\tilde{p}/p$ of the price $\tilde{p}$ at which the orders are executed over the previous quoted price p is solely a function of Ω and using the condition that it is not possible to make profits by repeatedly trading through a close circuit (i.e. by buying and selling with final net position equal to zero), Farmer has shown that the market impact function is an exponential

$$\frac{\tilde{p}}{p} = e^{\frac{\Omega}{L}} , \tag{16}$$

where L is the liquidity of the market. This equation is very intuitive : if $\Omega > 0$ (demand is larger than supply), the price increases. The reverse occurs if supply is larger than demand.

[1] A pure power law fit will not lock in on the true date of the crash, but insists on an earlier date than the last data point. This suggests that the behaviour of the price might be different in some sense in the last few weeks prior to the burst of the bubble.

The exponential dependence is interesting because it predicts that the time evolution of the logarithm of the price is given by the following equation

$$\frac{d \ln p}{dt} = \frac{1}{L} \Omega(t) \ , \tag{17}$$

where the net order size Ω over all traders is changing as a function of time so as to reflect the information flow in the market and the evolution of the traders' opinions and moods.

The simplest model is that the net order size Ω is a random uncorrelated noise. This is the limit where traders are heterogeneous, uncorrelated and the information flow occurs stochastically with no memory. This retrieves the celebrated random walk description, since the solution of (17) is that $\ln p$ performs a random walk.

The next level of description of imitative decreasing markets is to assume that there is a pessimistic spirit permeating the market such that the net order sizes $\Omega(t)$ at successive times are biased negatively, so that there is a trend for investors to slowly exit the market and cut down their overall positions. However, rather than assuming that the resulting dynamics of $\ln p$ is that of a (negatively) biased random walk, we posit that the negative "bearish" mood appears in bursts of pessimism interspersed in otherwise more neutral states characterised by $\langle \Omega \rangle \approx 0$ or even maybe slightly positive. Furthermore, it has often been observed that investors often behave in response to what appears to be "psychological" thresholds, like the Nikkei crossing 15.000. We will therefore introduce such thresholds in the dynamics, which will be found essential for the generation of log-periodicity. A possible mechanism for the existence of threshold and irreversible price behaviour is that, in a depression or in a negatively oriented market, cash available for investment progressively disappears to pay debt or in favour of other markets and, once realized from a sell, may no longer be available for continuing investment. We model this behaviour by introducing a quenched structure in the behaviour of the net order size. Finally, we note that prices are given in point units (0.01%) and are thus discrete. We can thus see the log-price variation as the evolution of a walker jumping from site to site along the axis measuring its level, where the mesh size correspond to one point.

The transitions from one log-price to another are performed according to the two possible scenarios :

1. with probability P, the log-price surely decreases to its neighbour below him. Furthermore, if this occurs, the price is unable to recover its previous level and remains for ever below. This is the quenched structure of the model taken as a naive approximation of the threshold effects in financial markets.

2. with probability $1 - P$, the log-price can either decrease with rate u or increase with rate v.

Thus formulated, the model is fully equivalent to the model studied by Bernasconi and Schneider [21] of a *frozen* random lattice constructed by choosing a given configuration of randomly distributed mixtures of the two bond species according to their respective average concentration P and $1 - P$. The exact solution of this problem has been given in [21] and shows very clearly nice log-periodic oscillations in the dependence of $\langle [\ln p]^2 \rangle$ as a function of time. A simple scaling argument has been shown [17, 22] to recover, in the limit $P \to 1$, the exact results according to which the prefered log-periodic ratio λ is given by

$$\lambda = \log(v/u) \tag{18}$$

and the local exponent is

$$\alpha = \log(1-P)/\log(u/v) \ . \tag{19}$$

The mechanism of log-periodicity in this model is the result of the interplay between the discreteness of the log-price increments and the existence of rare occurrence of time duration δ_t of optimisms in an otherwise "bearish" market, which occur with a probability exponentially small in δ_t as $(1-P)^{\delta_t}$ (see [17] for a general discussion and [10] for another application in strongly biased random walks in 3-d percolating networks).

This model provides a prediction that can be tested by comparing empirically determined values of λ and psychological moods of traders in the manner investigated by Shiller [23]. Indeed, expression (18) predicts that a larger λ corresponds to a larger ratio v/u. In words, if λ is larger from a devaluation than for a valuation bubble, this means that the relative strength of the optimistic phase in an otherwise pessimistic bearish market is stronger than the occasional pessimism in an otherwise speculative bullish bubble. (The reverse hold for smaller λ's.) The study be Shiller et al [24] confirm this and show that Japanese investors are considerable more optimistic about future trends in a decreasing Japanese stock market than in an increasing American stock market.

4 Conclusion

We have suggested that the analogy between financial crashes and critical phenomena can be extended to include a qualitative ensemble symmetry with respect to the time direction. In terms of the behaviour of the financial markets, this implies that power law decay of prices possibly decorated by log-periodic oscillations can be found. We find this indeed to be the case for the Nikkei stock market index and the price of gold futures. In both cases, a good agreement is obtained between the values of the power law exponent for the decay of the two fits and the values previously found for the power law increase in the market prices prior to large financial crashes. Furthermore, the strength of the oscillations compared to the leading behaviour indicates that they cannot be neglected. Indeed, the Nikkei case provides us with the strongest log-periodic signature identified in financial markets to this date, as clearly illustrated by the frequency analysis presented in figure 4 (compare the size of the peak to the background level).

However, the proposed time-symmetry operates only on an ensemble level, i.e., nowhere have we found log-periodicity for an increasing market followed by a decreasing market around the same critical time t_c[2]. Furthermore, the values found for log-frequencies and the corresponding prefered scale ratios λ are found less universal than for speculative bubbles ending in crashes. That λ differs for the Nikkei and the gold is not surprising. Gold plays a rather special role as a refuge in times of economic recession and strong pessimism, which was partly the reason for its bubble of the late seventies. Hence, the decay of the gold price in the early eighties does not coincide with a period of economic depression, but rather to the opposite where this refuge is abandoned in favour of more profitable investments in bullish times. In contrast, the decay of the Nikkei is the signature of an overall Japanese recession. It is thus not surprising that a market with a single commodity with such special characteristics displays more rapid oscillations with $\lambda \approx 1.9$ than that of a global stock market.

[2]Although the case of gold comes "close".

The results published in [11] for the 6 stock market and Forex bubbles indicates $\lambda \approx 2.5 \pm 0.3$ with the exception of the bubble of the US$ against the CHF, which gave $\lambda \approx 3.4$. However, the log-periodic signatures in this case were weaker than in the other 5 cases, i.e., only of ≈ 6 % of the leading behaviour. Hence, the larger value presumably results from the uncertainties in the parameter estimation.

In contrast, the log-periodic signatures seen for the decay of the Nikkei are the strongest obtained for the stock market as well as the Forex. This indicates that the larger value $\lambda \approx 3.4 - 3.6$ is not accidental and suggests that depressions belong to a different universality class for log-periodicity. This is maybe not surprising to find oscillations of a lower log-frequency for a depression than for a speculative bubble: according to our model, a larger λ corresponds to a larger ratio v/u, which means that the relative strength of the optimistic phase in an otherwise pessimistic bearish market is stronger than the occasional pessimism in an otherwise speculative bullish bubble. The large λ found for the depressed Nikkei then reflect the fundamental psychological asymmetry between optimism and pessimism.

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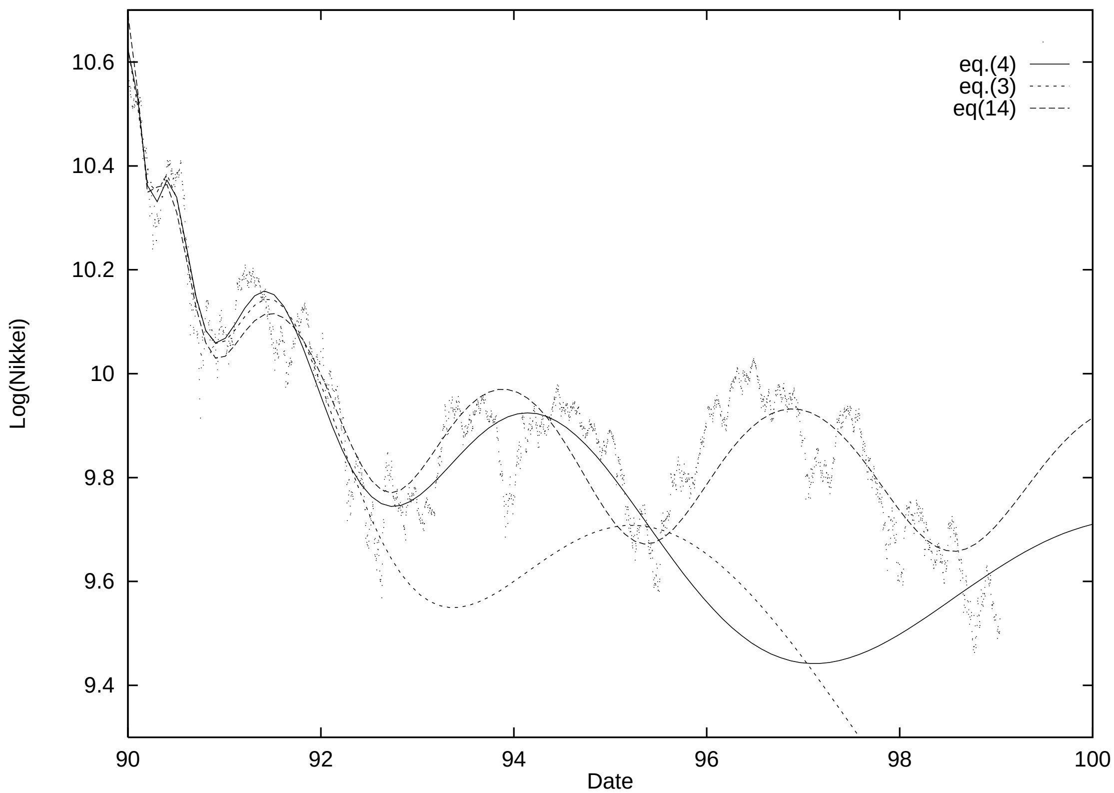

Figure 1: Natural logarithm of the Nikkei stock market index after the start of the decline 1. Jan 1990 until 31 dec. 1998. The lines are eq. (3) (dotted line) fitted over an interval of ≈ 2.6 years, eq. (4) (continuous line) over ≈ 5.5 years and eq. (14) (dashed line) over 9 years. The parameter values of the first fit of the Nikkei are $A \approx 10.7, B \approx -0.54, C \approx -0.11, \alpha \approx 0.47, t_c \approx 89.99, \phi \approx -0.86, \omega \approx 4.9$ for eq. (3). The parameter values of the second fit of the Nikkei are $A' \approx 10.8, B' \approx -.70, C' \approx -0.11, \alpha \approx 0.41, t_c \approx 89.97, \phi' \approx 0.14, \omega \approx 4.8, \Delta_t \approx 9.5, \Delta_\omega \approx 4.9$ for eq. (4). The third fit uses the entire time interval and is performed by adjusting only Δ_t, Δ_t', Δ_ω and Δ_ω', while α, t_c and ω are fixed at the values obtained from the previous fit. The values obtained for these four parameters are $\Delta_t \approx 4.34$, $\Delta_t' \approx 7.83$, $\Delta_\omega \approx -3.10$ and $\Delta_\omega' \approx 23.4$. Note that the values obtained for the two time scales Δ_t and Δ_t' confirms their ranking. This last fit predicts that the Nikkei should increase as the year 2000 is approached.

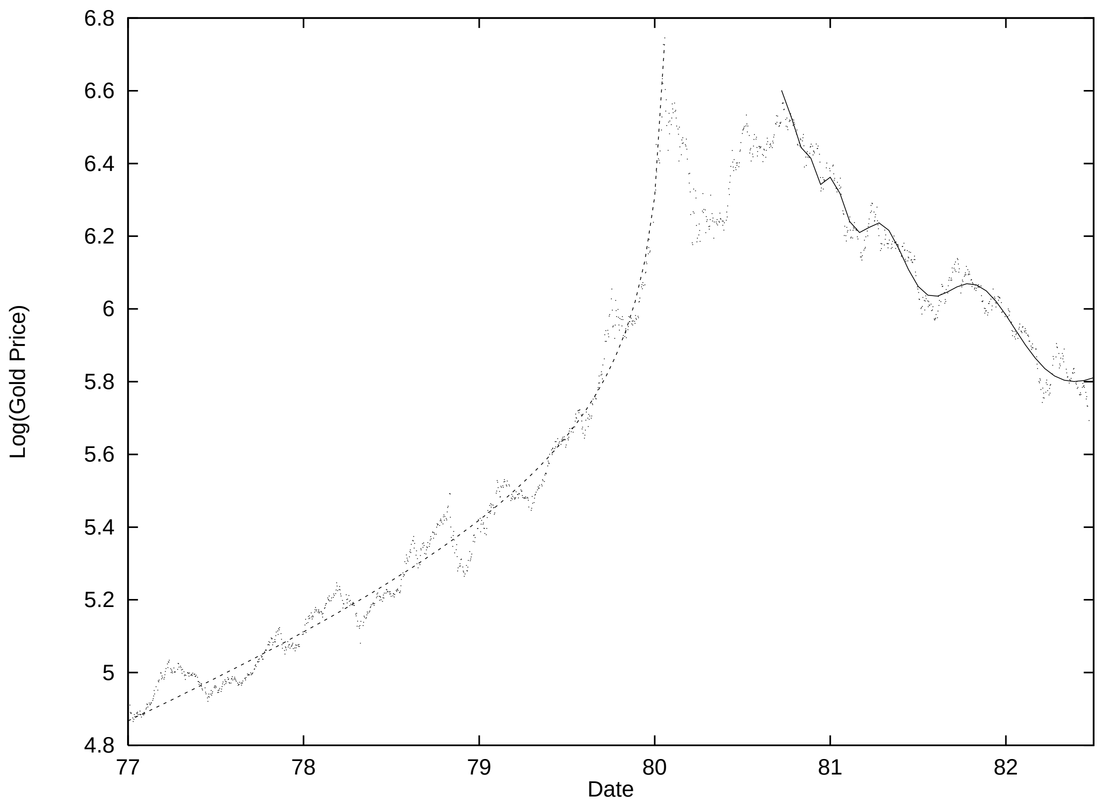

Figure 2: Natural logarithm of the gold 100 Oz Future price in US \$ after the decline of the price in the early eighties. The line after the peak is is eq. (3) fitted over an interval of ≈ 2 years. The parameter values of the fit are $A \approx 6.7, B \approx -0.69, C \approx 0.06, \alpha \approx 0.45, t_c \approx 80.69, \phi \approx 1.4, \omega \approx 9.8$. The line before the peak is eq. (3) fitted over an interval of ≈ 3 years. The parameter values of the fit are $A \approx 8.5, B \approx -111, C \approx -110, \alpha \approx 0.41, t_c \approx 80.08, \phi \approx -3.0, \omega \approx 0.05$

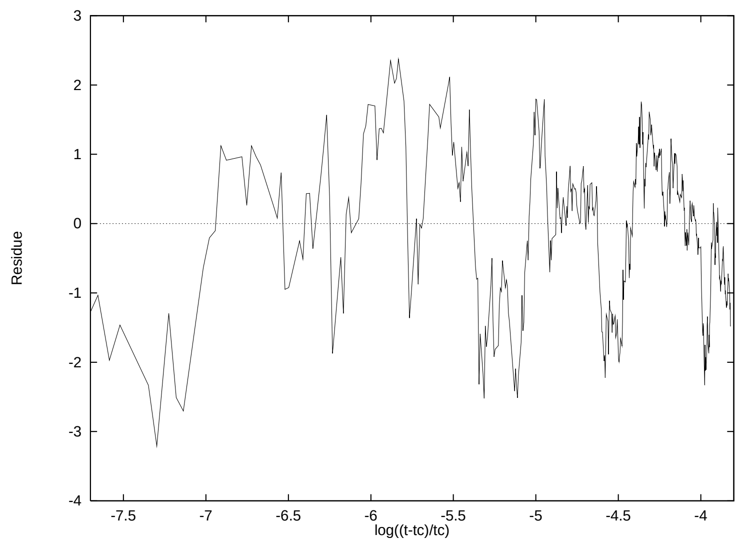

Figure 3: The residue as defined by the transformation (15) as a function of $\log\left(\frac{t-t_c}{t_c}\right)$ for the 1980 gold crash.

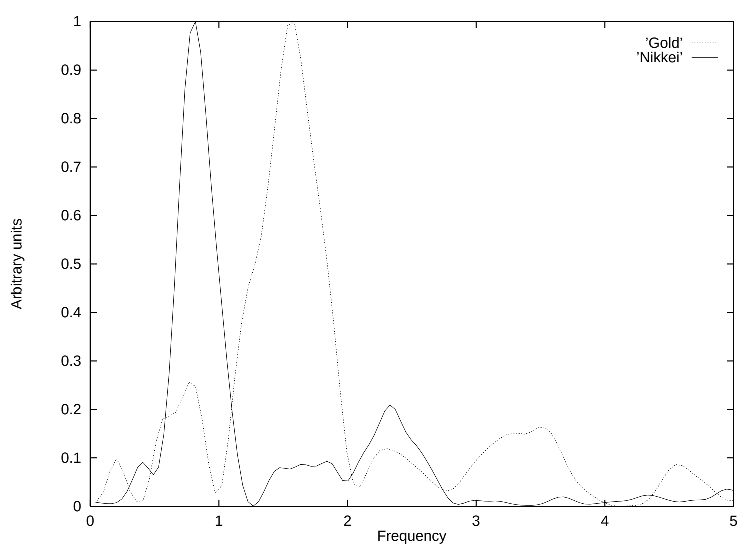

Figure 4: Lomb periodogram of the residue of the log(price) as definded by eq. (15) (as shown in figure 3 for gold) of the data shown in figures 1 and 2. For each periodogram, the significance of the peak should be estimated against the noise level.